Cantor Problem Set

Due: January 22, 2020

For each problem, construct a proof which either proves or disproves the proposition. Your answers must be typeset in latex, and if you use outside sources, you must cite your work. For full credit, you must attempt all of these problems. In addition to attempting all problems, you will be graded on your best 3 easier problems and your best 2 harder problems. (In other words, you must try them all, but you must succeed on at least 5 of them.) Bring a hardcopy to class for submission.

Easier Problems

1. Cantor's diagonal argument works with more than two symbols. For instance, instead of m and w, we could use 0,1,2,3,4,5,6,7,8,9 and still obtain the same result.
2. $\sqrt{2}$ is a non-algebraic number.
3. The set of all rational numbers are countable.
4. The set of all algebraic numbers is countable.
5. The set of all integers (positive and negative) is countable.
6. There are necessarily more rational numbers than irrational numbers.

Harder Problems

1. The set of all finite but unbounded sequences of m, w are countable.
2. Every transcendental (non-algebraic) number must contain all digits of its number base at least one time. For instance, in decimal, a transcendental number must have all digits from 0 to 9 somewhere in its infinite sequence.
3. The set of all prime numbers has the same cardinality as the set of all algebraic real numbers.
4. Besides infinite recursion of partitions and the diagonal argument, find a proof that the cardinality of natural numbers is strictly less than the cardinality of real numbers.
5. The set of infinities is itself uncountable.