

# 01 - Introduction and Math Preliminaries

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# Outline

- 1 Introduction to Compilers
- 2 S-Algol
- 3 Math Preliminaries

# What is a compiler?

A compiler ...

- verifies the validity of the source program.
- translate a source program into an object program.
- translates a source program without changing its semantic meaning.

# Program Stages

- Compile Time
  - Lexicographical Properties of the Program
  - Validation
  - Code Production
- Load Time
  - Loading and linking of shared libraries
  - Relocation of Code
- Run Time
  - Program Execution
  - Dynamic Behavior of the Program

# Phases of Compilation

- 1 Lexical Analysis
- 2 Syntax Analysis
- 3 Code Generation

# Lexical Analysis

- Analysis of **microsyntax** of a language.
- Breaking a program into tokens (aka basic symbols or lexemes)
- Not always trivial! (Context can alter tokens)  
Consider the following Fortran:

- DO 1 I=1, 12
- DO 1 I=1.12

# Syntax Analysis

- Analyzes the structure of the program.
- Validates structure. (i.e. Do { } match?)
- Results in a **parse tree** representation of the program.

# Code Generation

The code generator ...

- traverses the parse tree.
- generates object code as it descends the tree.
- optimizes object code.



# Recursive Descent Compiling

- Each major language structure has a corresponding recognizer routine.
- These methods call each other as needed.
- As the methods get called, they construct a parse tree.
- Errors are detected as the recognizers execute.
- Limited in scope to LL(1) languages.

# Language Properties

- ALGOL Inspired Language
- Sequence Level Scoping
- Types are Inferred at Declaration
- Vectors for Lists of Variables
- Structures
- Procedures
- Designed as a Teaching Language
- Powerful Enough for Systems Programming

# Variable Declarations

```
let x := 1           !has type int i.e.integer
let y := 2.7        !has type real
let switch := x<pi  !has type bool i.e. boolean
let name := "Bill"  !has type string
let e=2.71828       !real constant
let lbl := "here"   !has type cstring
```

# Structures

```
structre identifier(cstring name ;real val)  
let var := identifier("x", 2.14)
```

# Procedures

```
procedure count(cint s,e)
begin
  let x := s
  while x <= e do
  begin
    write x
    x := x + 1
  end
end
```

```
procedure convert(cint L,S,D->real)
  L+S/20+D/240
```

# Closure

- **Closure** - A sequence of objects which close a set of objects.
- We often speak of closure of grammars, languages, and sets.
- Computing closures reveal vital information about a language.
- We will get more formal with closures later.

# Alphabets and Languages

- An alphabet,  $A$ , is a finite set of symbols. For example:

$$A = \{a, b, c\}$$

- A language,  $L$ , is the set of sequences or strings over some alphabet. For example, we may have:

$$L = A \times A$$

- Expanding the above language yields:

$$L = \{aa, ab, ac, ba, bb, bc, ca, cb, cc\}$$

# Strings of Length $k$

- Often we wish to express arbitrarily long strings of alphabet  $A$ .
- Strings of length  $k$  are written as  $A^k$  where  $k$  represents the number of times the Cartesian product is applied to  $A$  and itself.
- For example:

$$A^3 = A \times A \times A$$

Represents all strings consisting of 3 symbols from  $A$ .

- $A^0$  is the empty string, we often give it the special symbol  $\lambda$



# Reflexive Transitive Closure

- Suppose we take  $A^k$  for all values  $k = [0 \dots \infty]$
- The union of every such set of sequences is called  $A^*$

$$A^* = \bigcup_{n=0}^{\infty} A^n$$

- $A^*$  is the reflexive transitive closure of  $A$
- Also known as the Kleene closure (after its definer, Stephen Kleene).
- $A^*$  is the language consisting of every possible string over the alphabet  $A$ .

# Transitive Closure

- Sometimes, we wish to exclude  $\lambda$  from the set of strings. This is the  $A^+$  closure.
- $A^+ = A^* - \lambda$
- Fully stated:

$$A^+ = \bigcup_{n=1}^{\infty} A^n$$

- This is called the Transitive Closure of  $A$

# Concatenation

- **Concatenation** is an operator which combines two strings.
- Concatenation (represented as a  $.$ ) defines an algebra over  $A^*$ .
- $\lambda$  is the unit element since  $\forall s \in A^*$ :

$$s.\lambda = \lambda.s = s$$

- Concatenation is associative, but not commutative.
- **Closure** is the smallest set containing a given basic set closed under certain operations.
- Hence  $A^*$  is the reflexive transitive closure of  $A$  under the operation of concatenation.
- Also,  $A^+$  is the transitive closure of  $A$  under the operation of concatenation.

# Languages and Strings

- If we think of a programming language,  $L$  is the set of all strings which form a valid program.
- Usually, we think of the alphabet of a programming language as being its individual lexemes.
- For almost all programming languages:

$$L \subset A^*$$

Where  $A$  is the alphabet of lexemes of the language.

- A compiler, is therefore formally stated as a device which, given a string  $s$ , a source language  $L$ , and an object language  $L'$ :

$\forall s \in A^*$ :

- 1 Decides  $s \stackrel{?}{\in} L$
- 2 Computes the function  $L \mapsto L'$  on  $s$