

Gödel Problem Set 2

Due: February 21, 2020

Answer each of the following problems. As always, you must attempt all of them. This time, you must complete one of the easy problems and one of the hard problems. Your answers to completed problems must be typeset in \LaTeX . Good attempts can be hand written, bad attempts must be delivered during lunch in Pearsons Hall in the form of interpretive dance.

1 Easy Problems that Look Hard

1. Create a formal system of proof which is complete. That is, create a formal system under which all true propositions are provable. Prove that your system does what you claim. (And please, no spoilers for anyone. Let everyone sweat it out.)
2. Suppose I have two proof systems α and β . Both systems are ω -consistent. Suppose I have proposition a which is undecidable in α , and a different proposition b which is undecidable in β . Suppose I can use some function to combine the two proof systems $\gamma = f(\alpha, \beta)$. Can the resultant system admit proofs of a or b and remain ω -consistent? Prove your answer.

2 Hard Problems that Look Impossible

1. Construct another C-undecidable number-theoretic proposition (where C is some ω -consistent formal system) that is not equivalent to the Gödel has given us. Prove that your proposition is undecidable under these constraints. Extra points will be awarded if your proposition can be decided outside of any formal system (as was Gödel's proposition.)
2. Fill in the outline of the proof presented in Gödel's proposition V. That is, present the formal proof which Gödel describes as not technically challenging, but terribly tedious. (Believe me, it is tedious.)